# Topological Properties of Solvable Graphs for Multiple Robot Motion Planning

Nam Nguyen ‘21, Bao Nguyen ‘21, Hiva Samadian

Colgate University

## Abstract

In multiple robot motion planning (MRMP), a Solvable or Reachable Graph for *m* robots ($SG^m$) is a graph that any arrangement of a specified number of mobile agents located on the graph’s vertices can be reached from any initial arrangement through agents’ moves along the graph’s edges. In this research, the topological properties of $SG^m$ were investigated to answer the problem of deciding whether a graph is Solvable for *m* robots, without explicitly solving it. We proved several necessary and sufficient conditions for several basic topologies. Results of this research, when is completed, can be used in designing transportation networks (e.g. railways, traffic roads, AGV routes, robotic workspaces, etc.) for multiple moving agents such as trains, vehicles, and robots.

## Some of Definitions

- A ***Mission*** is a set of moves used to reach a final configuration from an initial configuration.
- **A *Solvable Graph*** for *m* robots ($SG^m$) is a graph on which *any* Mission of *m* robots can be accomplished.
- ***Path($v_0$, $v_k$)*** represents a path between nodes $v_0$ and $v_k$.
- A **Cycle** is a Path($v_0$, $v_k$) in which $v_0 = v_k$
- A **Junction** is a non-cycle vertex with degree greater than or equal to 2
- ***X($r_i$, $g_i$)*** is the mission of moving $r_i$ to its goal $g_i$ while the positions of all other robots remain unchanged. When $g_i$ contains a robot $r_j$ the mission $X(r_i, g_i)$ is to exchange $r_i$ and $r_j$.
- A ***Hole*** is a vertex not occupied by any robot in an initial configuration. The total number of holes in a graph is H.

In a graph *G=(V,E)* with *H* holes:

- **The *Cycle Influence Zone* of a cycle *O*** is the subgraph $G' = (V', E')$ of G in which:

  $V'$={v∈V: ∃ a vertex $v' \in O$ and a path $P=Path(v, v')$ with $|P| \leq H$ s.t. none of the edges of *P* belong to a cycle *other than O*}

  $E'$={$e \in E$: *e* is an edge on a path $P=Path(v, v')$ for a v∈V′}
- **The *Junction Influence Zone* of a Junction *J*** is the subgraph $G' = (V', E')$ of G in which:

  $V'$={v∈V: ∃ a path $P=Path(v,J)$ with $|P| \leq H-1$ s.t. there is no cycle-edge on *P*}

  $E'$={ e∈E: *e* is an edge on a path $P=Path(v,J)$ for a v∈V′}
- **A junction influence zone ($IZ_J$)** in a graph is a junction together with its influence zone.
- **A cycle influence zone ($IZ_O$)** in a graph is a cycle together with its influence zone.
- **Adjacency**

Two junctions J1 and J2 are considered Adjacent if they are not connected by a path that has a cycle-node or an outsider junction node on it.

Two Cycles O1 and O2 are considered Adjacent if they are not connected by a path that has a junction or an outsider cycle-node on it.

Two Cycle O and junction J are considered Adjacent if they are not connected by a path that has an outsider junction or an outsider cycle-node on it.

Two IZs are Adjacent IZs if their junctions/cycles are Adjacent.

- **Interconnectedness**

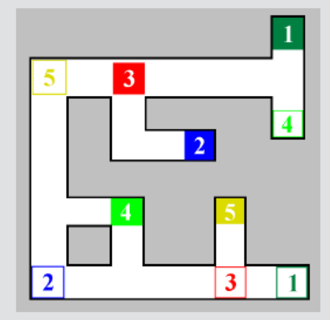
Two Influence Zones ($IZ^p$ and $IZ^q$) are *Interconnected* if they are Adjacent and for their junctions/cycles the following conditions are valid:

| | |
|---|---|
| Dist ($J^p$, $J^q$ ) ≤H-2 | if both IZs are junction IZs |
| Dist ($O^p$, $O^q$ ) ≤H | if both IZs are cycle IZs |
| Dist ($J^p$, $O^q$ ) ≤H-1 | if $IZ^p$ is $IZ_J$ and $IZ^q$ is $IZ_O$ |

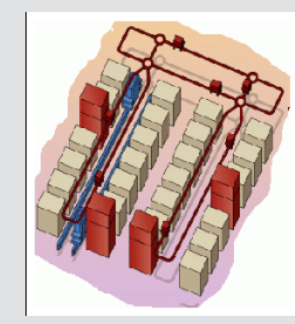
## Our Approach

**Setup:**

- Given a workspace and configuration of robots, construct a graph whose vertices include the robots’ initial positions and destinations and whose edges pass through free spaces.



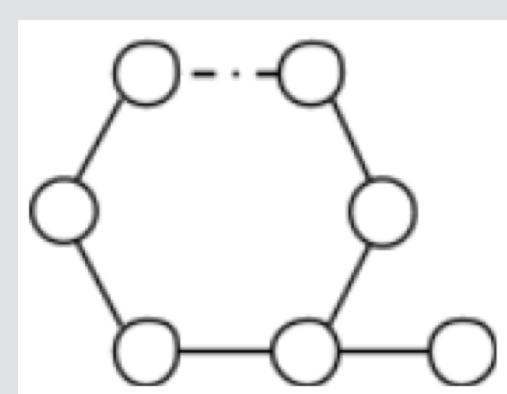
*Graphic Representation*



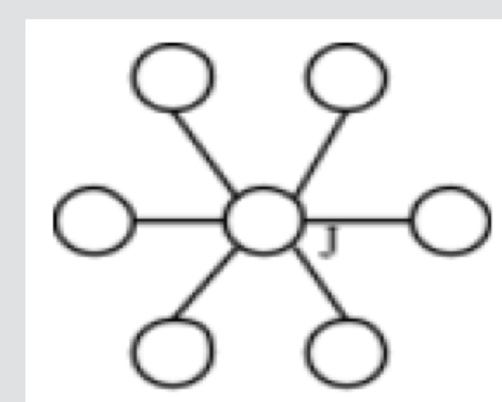
*Actual Workspace*

**Solvability of single Influence Zones (IZ)**

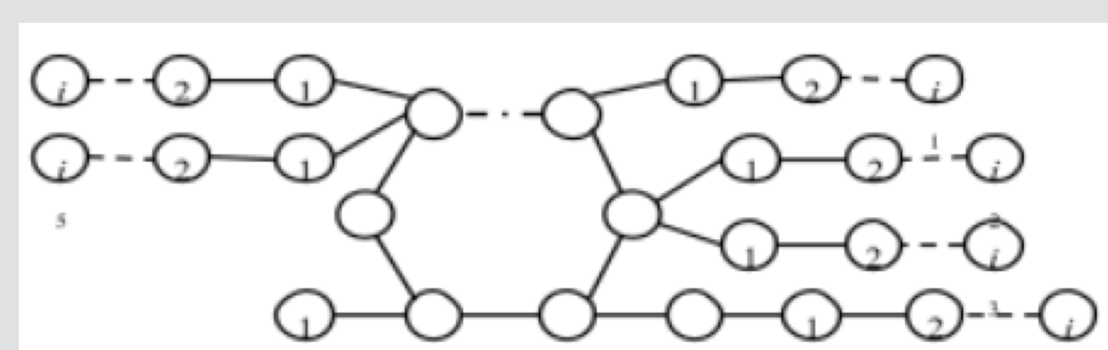
- Because of the effect of Junctions and Cycles on solvability, we defined two concepts called Junction Influence Zone and Cycle Influence Zone.
- To determine the solvability of an Influence Zone, we first defined four basic topological structures in graphs as follows:



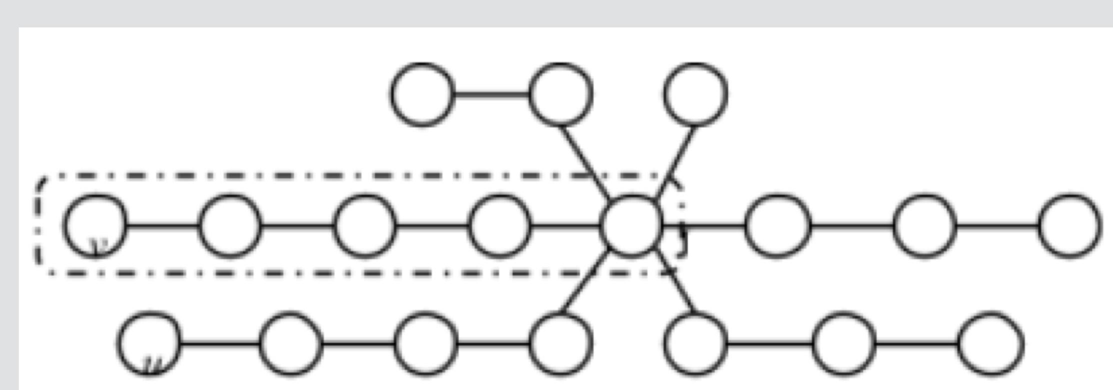
Basic Unicycle



Star



Extended Unicycle



Extended Star

- Determine the conditions for solvability of the above graphs
- Determine the conditions for solvability of single Influence Zones.

**Solvability of general graphs**

- Partition these graphs into a number of subgraphs, all of which have a single Influence Zone.
- Determine the conditions for solvability of these general graphs.

## Experiment & Results

- A graph is solvable if and only if every $X(r_i, g_i)$ is possible for each initial configuration.
- A Basic Unicycle $BU^k$ is solvable *iff* $H \geq 1$.
- An Extended Unicycle $EU^k$ is solvable *iff* $H \geq \|S\|$.
- Every $IZ_O$ in every graph *G* is solvable *iff* all holes of *G* are in $IZ_O$
- A Star $Star^k$ is Solvable *iff* $H \geq 2$.
- An Extended Star $ES^K$ is solvable *iff* $H \geq \|S\| + 1$.
- Every $IZ_J$ in every graph *G* is solvable *iff* all holes of *G* are in $IZ_J$
- For any two Adjacent *IZ*s, any robot located in one *IZ* can access any vertex in the other *IZ iff* the two Zones are Interconnected.

**Experiment: Java program to generate graphs and test lemmas**

- Java program has a Graph class with Nodes and Edges classes to represent the nodes and edges of the graph.
- There is a Robot class to keep track of the robot, which node they reside and the position they intend to head towards.
- Upon entering a type of graph and number of vertices, the program will generate the desired type of graph for the user. There is also a class Branch to represent the different branches.
- User can provide a number of robots to be distributed on the graph. The program will generate an arbitrary initial configuration and final configuration of the robot.
- By specifying the lemma (i.e. one of the results above), the user can test whether the algorithm described in the proof of the lemma works for the robot.

## Conclusion and Future Work

The paper details the proofs for topological properties of a Solvable Graph for *m* robots ($SG^m$). Examining four types of graphs: Basic Unicycle, Star, Extended Unicycle and Extended Star, the paper determines the conditions for solvability of these graphs. These results will aid in the process of Multiple Robot Motion Planning.

Future work will include examining the topological properties of Purely Cyclic Graphs and Partially Solvable Graphs. Additional work would also be to invest the properties of Minimal Solvable Graphs (MSG), which are the smallest graphs in the number of vertices that can satisfy the feasibility conditions for multiple robot motion planning. Also, our assumptions for future works may involve all moves to be concurrent and find the minimum sequence of moves to reach the final configuration with different robot velocities.

## Selected References

E. Masehian and A.H. Nejad, “Solvability of multi robot motion planning problems on trees”, in *Proc. International Conference on Intelligent Robots and Systems (IROS) 2009*, St. Louis, USA, pp. 5936-5941.

*J.C. Latombe, Robot Motion Planning, Kluwer Acad. Pub., London, 1991.*

## Acknowledgment

We thank the Holden Endowment Fund for financially supporting this research.